Free-Standing Mathematics Qualification
June 2006
Advanced Level

MODELLING WITH CALCULUS **6992/2**
Unit 12

Thursday 18 May 2006 1.30 pm to 3.00 pm

For this paper you must have:
- an 8-page answer book
- a calculator
- a clean copy of the Data Sheet (enclosed)

Time allowed: 1 hour 30 minutes

Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is 6992/2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.
- The **final** answer to questions requiring the use of a calculator should normally be given to three significant figures.
- You may **not** refer to the copy of the Data Sheet that was available prior to this examination. A clean copy is available for your use.

Information
- The maximum mark for this paper is 60.
- The marks for questions are shown in brackets.

There are no questions printed on this page

SECTION A

Answer **all** questions.

Use **Tennis** *on page 2 of the Data Sheet.*

1 Maria hits a ball while playing tennis.

The vertical height of the ball, y metres, above A, the point at which it was hit, is given by

$$y = 4t - 5t^2$$

where t is the time in seconds after the ball is hit.

(a) (i) Find the height of the ball above A when $t = 0.5$. *(1 mark)*

(ii) Find the height of the ball above A when $t = 1$.
Interpret your answer. *(2 marks)*

(b) Find $\frac{dy}{dt}$, the vertical velocity of the ball, in metres per second. *(2 marks)*

(c) Find t when $\frac{dy}{dt} = 0$. *(2 marks)*

(d) Hence predict the maximum vertical height of the ball above A. *(2 marks)*

(e) (i) Find $\frac{d^2y}{dt^2}$. *(1 mark)*

(ii) Hence state how this value confirms that the answer to part (d) is the maximum height and not the minimum. *(1 mark)*

(f) Maria hits the ball when it is 2.4 metres above the level of the horizontal ground.

Find the time when the ball hits the ground. *(4 marks)*

Turn over for the next question

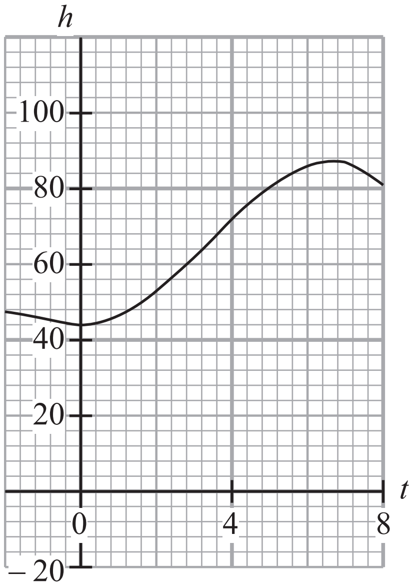
SECTION B

Answer **all** questions.

Use **Tides** *on page 2 of the Data Sheet.*

2 At 8 am, the time, t hours, was taken to be 0.
For values of t from 0 to 8, the height of the water, h centimetres, may be modelled by the function

$$h = 43 + 3t^2 - 0.3t^3$$



(a) **Using the model** $h = 43 + 3t^2 - 0.3t^3$ **and calculus**, find the maximum height of the water. *(6 marks)*

(b) State the time when the height of the water is at a minimum. *(1 mark)*

(c) The mean height of the water, $\bar{h}$, during the first 8 hours is given by

$$\bar{h} = \frac{\int_0^8 (43 + 3t^2 - 0.3t^3)\,\mathrm{d}t}{8}$$

(i) Use the trapezium rule with four strips to find an estimate for the mean height of the water during these eight hours. *(5 marks)*

(ii) Use integration to find the exact value of $\bar{h}$. *(4 marks)*

(d) When $t = 6$, the actual height of the water was 90 cm.

Find the percentage error, when $t = 6$, in using the model to predict the height of the water. *(3 marks)*

(e) Find the value of t when $\frac{\mathrm{d}^2h}{\mathrm{d}t^2}$ is zero. Interpret what is happening at this time. *(4 marks)*

Turn over for the next question

SECTION C

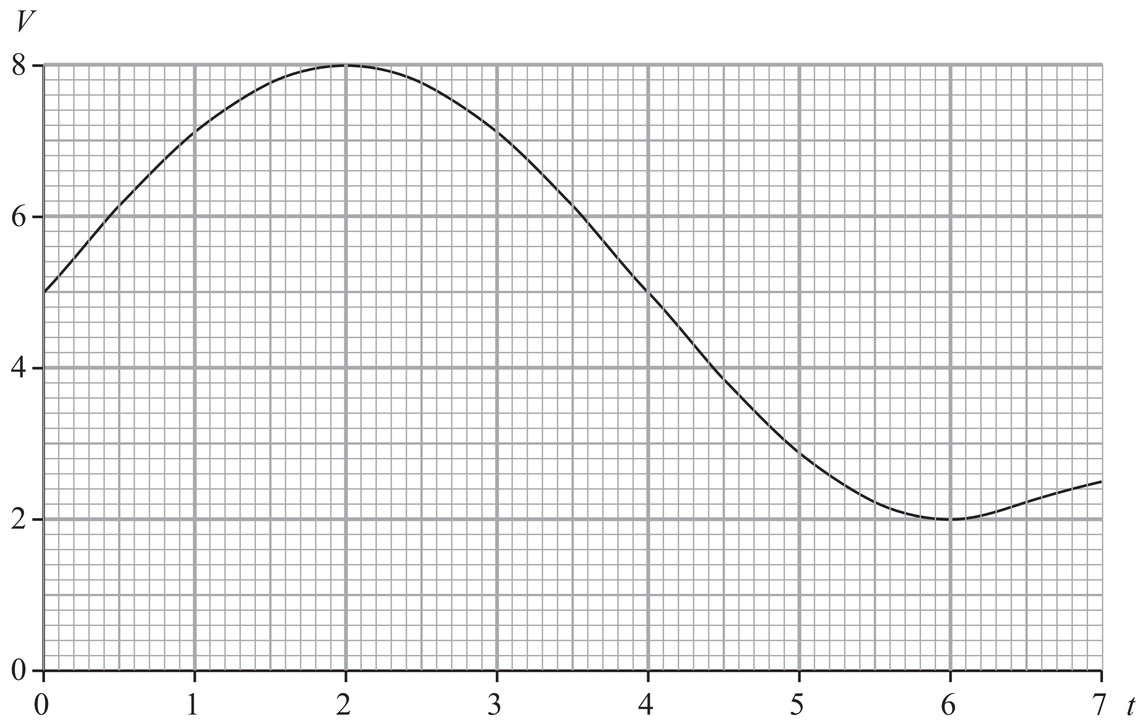
Answer **all** questions.

Use **Value of Lisbon Logistics shares** *on page 3 of the Data Sheet.*

3 The value of Lisbon Logistics shares, £V, during the first seven months of the year 2005, may be modelled by the function

$$V = 5 + 3\sin\frac{\pi}{4}t$$

where t is the time in months after 1 January 2005.
The graph of this equation for $0 \le t \le 7$ is shown below.



(a) State the value predicted by the model when:

(i) $t = 0$; *(1 mark)*

(ii) $t = 1$. *(1 mark)*

(b) (i) Find an expression for $\frac{dV}{dt}$. *(3 marks)*

(ii) Show that a highest point predicted by the model is when $t = 2$. *(2 marks)*

(c) (i) State the maximum value of $\frac{dV}{dt}$. *(1 mark)*

(ii) Explain what is happening when $\frac{dV}{dt}$ has this value. *(1 mark)*

SECTION D

Answer **all** questions.

Use **Temperature** *on page 4 of the Data Sheet.*

4 A bottle of milk is taken out of a fridge at 2 ºC and placed in a room at a temperature of 20 ºC.

After t minutes, the temperature, c (in ºC), satisfies the equation

$$\frac{\mathrm{d}c}{\mathrm{d}t} = \tfrac{1}{40}(20 - c)$$

(a) (i) Find $\frac{\mathrm{d}c}{\mathrm{d}t}$ when $c = 5$. *(1 mark)*

(ii) Interpret this value. *(1 mark)*

(b) Show that $\tfrac{1}{40}t = \ln \frac{18}{20 - c}$. *(4 marks)*

(c) Rearrange this equation to give c in terms of t. *(3 marks)*

(d) When $t = 10$, find the temperature of the milk. *(2 marks)*

(e) (i) State the value which c approaches as t becomes very large. *(1 mark)*

(ii) State the value of $\frac{\mathrm{d}c}{\mathrm{d}t}$ as t becomes very large. *(1 mark)*

END OF QUESTIONS

There are no questions printed on this page