General Certificate of Education
June 2005
Advanced Subsidiary Examination

MATHEMATICS **MPC2**
Unit Pure Core 2

Tuesday 7 June 2005 Afternoon Session

In addition to this paper you will require:
- an 8-page answer book;
- the **blue** AQA booklet of formulae and statistical tables.

You **may** use a graphics calculator.

Time allowed: 1 hour 30 minutes

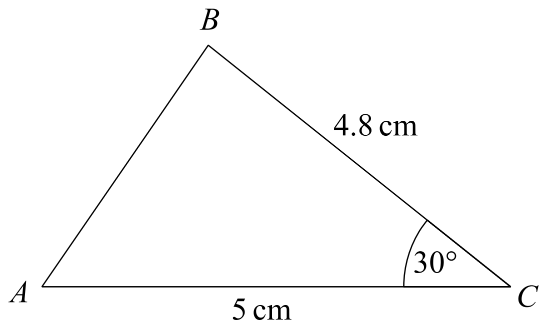
Instructions
- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- All necessary working should be shown; otherwise marks for method may be lost.

Information
- The maximum mark for this paper is 75.
- Mark allocations are shown in brackets.

Advice
- Unless stated otherwise, formulae may be quoted, without proof, from the booklet.

Answer **all** questions.

1 The diagram shows a triangle ABC.



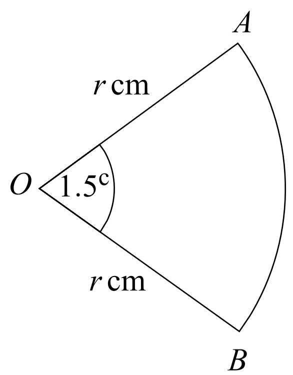
The lengths of AC and BC are 5 cm and 4.8 cm respectively.

The size of the angle BCA is 30°.

(a) Calculate the area of the triangle ABC. *(2 marks)*

(b) Calculate the length of AB, giving your answer to three significant figures. *(3 marks)*

2 The diagram shows a sector OAB of a circle with centre O and radius r cm.



The angle AOB is 1.5 radians. The perimeter of the sector is 56 cm.

(a) Show that $r = 16$. *(3 marks)*

(b) Find the area of the sector. *(2 marks)*

3 The nth term of an arithmetic sequence is u_n, where

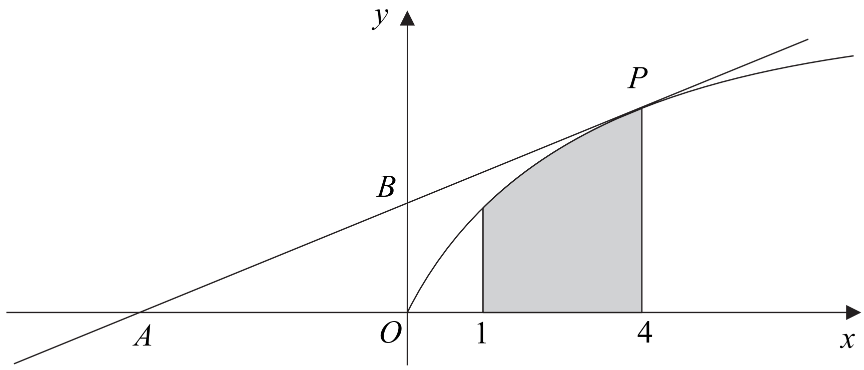
$$u_n = 90 - 3n$$

(a) Find the value of u_1 and the value of u_2. *(2 marks)*

(b) Write down the common difference of the arithmetic sequence. *(1 mark)*

(c) Given that $\sum_{n=1}^{k} u_n = 0$, find the value of k. *(3 marks)*

4 The diagram shows a curve C with equation $y = \sqrt{x}$. The point O is the origin $(0, 0)$.



The region bounded by the curve C, the x-axis and the vertical lines $x = 1$ and $x = 4$ is shown shaded in the diagram.

(a) (i) Write $\sqrt{x}$ in the form x^p, where p is a constant. *(1 mark)*

(ii) Find $\int \sqrt{x}\,dx$. *(2 marks)*

(iii) Hence find the area of the shaded region. *(3 marks)*

(b) The point on C for which $x = 4$ is P. The tangent to C at the point P intersects the x-axis and the y-axis at the points A and B respectively.

(i) Find an equation for the tangent to the curve C at the point P. *(4 marks)*

(ii) Find the area of the triangle AOB. *(3 marks)*

(c) Describe the single geometrical transformation by which the curve with equation $y = \sqrt{x-1}$ can be obtained from the curve C. *(2 marks)*

(d) Use the trapezium rule with four ordinates (three strips) to find an approximation for $\int_1^4 \sqrt{x-1}\,dx$, giving your answer to three significant figures. *(4 marks)*

Turn over ▶

5 The sum to infinity of a geometric series is four times the first term of the series.

(a) Show that the common ratio, r, of the geometric series is $\frac{3}{4}$. *(3 marks)*

(b) The first term of the geometric series is 48. Find the sum of the first 10 terms of the series, giving your answer to four decimal places. *(2 marks)*

(c) The nth term of the geometric series is u_n and the $(2n)$th term of the series is u_{2n}.

(i) Write u_n and u_{2n} in terms of n. *(2 marks)*

(ii) Hence show that $\log_{10}(u_n) - \log_{10}(u_{2n}) = n\log_{10}\left(\frac{4}{3}\right)$. *(3 marks)*

(iii) Hence show that the value of

$$\log_{10}\left(\frac{u_{100}}{u_{200}}\right)$$

is 12.5 correct to three significant figures. *(2 marks)*

6 (a) Using the binomial expansion, or otherwise, express $(1+x)^4$ in ascending powers of x. *(3 marks)*

(b) (i) Hence show that $(1+\sqrt{5})^4 = 56 + 24\sqrt{5}$. *(3 marks)*

(ii) Hence show that $\log_2(1+\sqrt{5})^4 = k + \log_2(7+3\sqrt{5})$, where k is an integer. *(3 marks)*

7 A curve is defined, for $x > 0$, by the equation $y = \text{f}(x)$, where

$$\text{f}(x) = \frac{x^8 - 1}{x^3}$$

(a) Express $\dfrac{x^8 - 1}{x^3}$ in the form $x^p - x^q$, where p and q are integers. *(2 marks)*

(b) (i) Hence differentiate $\text{f}(x)$ to find $\text{f}'(x)$. *(2 marks)*

(ii) Hence show that f is an increasing function. *(2 marks)*

(c) Find the gradient of the normal to the curve at the point $(1, 0)$. *(3 marks)*

8 (a) (i) Show that the equation

$$4\tan\theta\sin\theta = 15$$

can be written as

$$4\sin^2\theta = 15\cos\theta$$ *(1 mark)*

(ii) Use an appropriate identity to show that the equation

$$4\sin^2\theta = 15\cos\theta$$

can be written as

$$4\cos^2\theta + 15\cos\theta - 4 = 0$$ *(2 marks)*

(b) (i) Solve the equation $4c^2 + 15c - 4 = 0$. *(2 marks)*

(ii) Hence explain why the only value of $\cos\theta$ which satisfies the equation

$$4\cos^2\theta + 15\cos\theta - 4 = 0$$

is $\cos\theta = \frac{1}{4}$. *(1 mark)*

(iii) Hence solve the equation $4\tan\theta\sin\theta = 15$ giving all solutions to the nearest 0.1° in the interval $0° \leqslant \theta \leqslant 360°$. *(2 marks)*

(c) **Write down** all the values of x in the interval $0° \leqslant x \leqslant 90°$ for which

$$4\tan 4x\sin 4x = 15$$

giving your answers to the nearest degree. *(2 marks)*

END OF QUESTIONS

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE

THERE ARE NO QUESTIONS PRINTED ON THIS PAGE